THE USE OF GEOMETRIC DIAGRAMS IN THE PRIMARY SCHOOL

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Diagrams are a form of communication that are particularly useful for conveying geometric ideas. Children experience difficulty with diagram interpretation in geometry and also with the interpretation of graphics in geography. The similarity of children's difficulties in geometry and geography led to an examination of a model of levels of mastery of representation (Liben & Downs, 1991) based on geography, to determine its applicability for mathematics. Diagram interpretation was explored through observation of eight babies' behaviour with a three-dimensional shape and a corresponding diagram, and interviews with eight Grade 2 children and eight Grade 5 children. This study provides evidence of differing levels of children's understanding of geometric diagrams. Although the results support only two of the four levels of Liben and Downs' (1991) model, a refined five level model is proposed. The results indicate that the interpretation of diagrams may be a constraint to effective communication in geometry.

Geometry is to receive increased attention in primary mathematics education in accordance with curriculum initiatives for the twenty-first century (e.g., National Council of Teachers of Mathematics [NCTM], 1989). The emphasis in geometry has shifted from simply naming geometric shapes and memorizing geometric facts, to understanding the properties of geometric shapes and developing spatial sense (NCTM, 1989). Children find geometry difficult, with areas of concern in geometry including prototypes, definitions, language use and logical reasoning (Fuys & Geddes, 1984; Wilson, 1990). Hence, the new directions proposed for geometry pose a challenge for educators!

The concern about geometry, together with the current emphasis on the importance of communication in mathematics [NCTM, 1989], highlights the urgent need for research on geometric diagrams as a representational mode. Intrinsic in the use of a geometric diagram as a form of communication, is the assumption of shared meaning between diagram encoders (teachers, text illustrators and researchers) and diagram decoders (children). Diagrams are an important form of communication in geometry, as they represent knowledge about geometric shapes (Yerushalmy & Chazan, 1990). Therefore, children who are not diagrammatically literate are essentially denied access to the geometric communication.

DIAGRAMS IN GEOMETRY

The use of diagrams in geometry necessitates an understanding of geometric conventions, for example, the use of parallel perspective and the dual use of a single shape, as a specific example of a class and as a symbol for a class of shapes. Parzysz (1991) suggested that geometric conventions should be made explicit and that the geometric diagram should be given the status of a technical drawing.

The use of a specific shape, that embodies the attributes of a class, provides students with a protypal example and has a strong influence on concept development (Hershkowitz, 1989). However, the use of prototypes is characteristic of the "incompleteness" of the presentation of geometric concepts (Hershkowitz, 1989), consequently students can be misled about the critical attributes of a class. Nevertheless, as the students' understanding of the concept develops, the prototype becomes more flexible (Wilson, 1990).

GRAPHIC COMMUNICATION

The flexibility of graphic communication can lead to difficulty in interpreting geometric diagrams. The difficulty occurs when diagrams of two- and three-dimensional shapes are interpreted pictorially. A diagrams of a two-dimensional shape is simply a re-presentations of the referent and can be interpreted pictorially, because the diagram is perceptually "close" to the referent (Parzysz, 1988). However, there is a perceptual "distance" between a three-dimensional shape and its abstract diagrammatic representation (Parzysz, 1988). If diagrams of three-dimensional shapes are interpreted pictorially, there may be some confusion between the characteristics of the diagram and the characteristics of the referent.

The loss of information caused by the reduction of a three-dimensional referent to a two-dimensional diagram, can result in a conflict between what is known about a shape and what is actually seen on a diagram (Parzysz, 1988). Hence, perceptual flexibility is needed in the interpretation of diagrams of three-dimensional shapes (McKim, 1980). Students need to differentiate between diagrams of two-and three-dimensional shapes and "shift gears" for accurate interpretation.

A MODEL OF LEVELS OF MASTERY OF REPRESENTATION

Liben and Downs (1991) have proposed a model of levels of mastery of representation based on students' interpretation of the relationship between the graphic and the referent in map reading. The failure of students in geography to separate the characteristics of a graphic symbol from its physical referent (Liben & Downs, 1991) corresponds to the difficulties that may occur in geometry, if students fail to differentiate between two- and three-dimensional geometric referents. The aim of this study was to test the applicability of the model of levels of mastery of representation (Liben & Downs, 1991) for mathematics and if necessary, to refine the model to suit geometry. Table 1 shows the four level model of levels of mastery of representation (Liben & Downs, 1991).

Table 1

A model of levels of mastery of representation

Level 1 - Syncretism: The child reacts to the representation as if it were the physical referent.
Level 2 - Syncretic Representation: The child understands that the symbol "stands for" a referent, but confuses the referent characteristics with the pictorial characteristics.
Level 3 - Naive Conventional Representation: The child has a novice understanding of the relationship between a symbol and a referent.
Level 4 - Meta-Representation: The child understands the varieties of representation possible for a referent and the uses of the representation.

METHOD

Subjects

Twenty-four children participated in the study. Eight babies between eight and thirteen months, eight children in Grade 2 and eight children in Grade 5. The school children were selected by their class teachers as representative of the range of mathematical abilities within each grade level. They attended a state school in a middle socio-economic area of Brisbane. The babies were residents of the same suburb as the school children.

Instruments

Two types of instruments were used in this study. The first instrument was used with the babies and comprised a set of manipulatives, two floor mats and a foam-filled cube. The floor mats were plain blue and one metre square. The foam-filled cube had a side of twenty centimetres and was made from two squares of red, yellow and green fabric. The foam-filled cube was placed on one mat and a diagram of the cube was sewn onto the other mat. The coloured faces on the diagram corresponded to the coloured faces on the foam-filled cube.

The second instrument consisted of two sets of diagrams and was used with the Grade 2 and Grade 5 children. The first set of diagrams were of cubes and the second set of diagrams were of rectangular prisms. Each set of diagrams showed sixteen representations of the cube or rectangular prism based on variations with respect to size, perspective and orientation. The diagrams were randomly placed on the pages.

Procedure

The babies were placed on one of the mats and their behaviour observed. The mat was then removed and the procedure was repeated with the other mat. The order in which the babies were presented with the mats was counterbalanced. The babies' behaviour was video-taped for later analysis.

The Grade 2 and Grade 5 children were presented with the two sets of diagrams on a one-to-one basis during a single session. The diagrams of the cubes were presented first, followed by the diagrams of the rectangular prisms. The children were shown a wooden cube in various orientations and asked to tick the diagrams that showed a cube and cross the diagrams that didn't show a cube. As the children ticked and crossed the diagrams they were asked to explain their responses. A similar procedure occurred with the diagrams of the rectangular prisms, except that the children were shown two wooden rectangular prisms, one which had square faces and one which had no square faces. The responses of the children were video-taped for later analysis.

Data Analysis

The video-tapes of the babies' behaviour on the mats were analysed for the pattern of behaviour that they exhibited towards the cube and the diagram of the cube. The children's responses to the diagrams were analysed through the explanations given for the acceptance or rejection of the diagrams as cubes or rectangular prisms.

RESULTS AND DISCUSSION

The results of the observation of the babies and the interviews with the children on the diagram interpretation task confirmed that there are different levels of performance in the interpretation of diagrams of three-dimensional geometric shapes. However, the research findings refuted Liben and Downs'(1991) model of representation as a whole.

Syncretism was rejected as a level of mastery of representation as there was no evidence to suggest that the babies behaved similarly towards the cube and the diagram of the cube. Although the behaviour of the babies differed, each baby's behaviour with the cube was more active than his/her behaviour with the diagram of the cube.

The earliest level of interpretation identified in this study was exhibited by children who recognized that the diagram stood for something else, but were hindered in their interpretation by their lack of understanding of geometric shapes and/or diagrams. For example, Natalie, a Grade 5 student, appeared to have limited geometric knowledge. Although two rectangular prisms were shown to Natalie, her description and subsequent identification of a possible rectangular prism related solely to the "protypal rectangular prism" she had described earlier in the interview.

Natalie: It's got six sides and two of them are squares and four of them are rectangles (Natalie's earlier description of a rectangular prism).

Natalie: That doesn't look like a square (Natalie pointed to a side face on the diagram of a rectangular prism).

Bruce, also a Grade 5 student, was hindered in his interpretation by his limited understanding of diagrammatic representation, when he rejected a diagram of a rectangular prism..

Bruce: This side here should have a diagonal line.

Children like Natalie and Bruce, appear to need an **induction to representations**, where they have the opportunity to develop their geometric knowledge, including knowledge of geometric shapes and geometric conventions, in order to correctly interpret geometric diagrams.

Although syncretism was rejected in favour of induction to representations, as the initial level of mastery of representations (Liben & Downs, 1991), there was evidence to support the inclusion of **syncretic representation** and **naive conventional representation** as subsequent levels. Syncretic representation was exhibited by children like Anne, a Grade 2 student, who confused the properties of the referent and the properties of its representation. Anne rejected a diagram as a cube because of the pictorial qualities of the diagram.

Anne: It looks like a diamond.

Naive conventional representation was demonstrated by children who were able to correctly interpret diagrams, but showed some concern about diagrammatic conventions. For example, although Elise, a Grade 5 student, correctly identified a diagram as a cube, she appeared worried about the diagram and there appeared to be a conflict between what she saw and what she knew about a cube.

Elise: It's got squares.

Interviewer: What's puzzling you?

Elise: Those don't go straight (Elise pointed to the oblique lines on the diagram).

Children who had overcome the perceptual/conceptual conflict, but would not be considered expert in diagram interpretation, were considered to be at a level of **functional representation**. They were becoming aware of the conventions, limitations and ambiguity of diagrams. For example, Rebecca, a Grade 5 student, showed some understanding of ambiguity of diagrams.

Rebecca: Well in one case it might be a rectangular prism and in another case it might be a cube. You can't tell from the drawing.

Children at the level of functional representation have scope to improve their understanding of ambiguity in diagram interpretation. A further level of mastery of representations would be the level where there is an understanding of the multiplicity of representations possible for a single geometric shape and the multitude of geometric shapes that are the possible sources of a single representation, hence **multi-representation and multi-source**. This level enlarges upon **meta-representation** (Liben & Downs, 1991), which does not include multiple sources of a representation.

Table 2 shows a comparison between Liben and Downs'(1991) model of levels of mastery of representation and a model of representation of three-dimensional geometric shapes. Blank spaces on the table indicate that there is no comparable level on the other model.

Table 2

A Comparison of Models of Representation

Model of Levels of Mastery of Representation (Liben & Downs, 1991)	Model of Representation of Three-Dimensional Geometric Shapes
1. Syncretism	
	1. Induction to Representation
2. Syncretic Representation	2. Syncretic Representation
3. Naive Conventional Representation	3. Naive Conventional Representation
	4. Functional Representation
4. Meta-Representation	5. Multi-Representation and Multi-Source

CONCLUSION

Although the model of representation of three-dimensional geometric shapes proposed in Table 2 is based on a small sample and needs testing, the model is useful in raising the awareness of educators about the necessity for diagram literacy in geometry, and the various aspects of diagram literacy. The levels of literacy cover the transition from a diagram of a three-dimensional shape being interpreted as a re-presentation of a referent, to a diagram of a three-dimensional shape being interpreted as a representation of a referent.

The concern about geometry expressed by the NCTM (1989) presents a challenge to educators to ensure that all students are literate in diagram interpretation. Just as students who do not understand English are disadvantaged when instruction is in English, so to are students of geometry who are not literate in diagrammatic communication. Hence, a constraint to students' learning is the "mother tongue" of geometry, diagrammatic communication. In order to achieve the goal of mathematical literacy in geometry, the importance of visual communication must be acknowledged by educators.

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